

Covariant Classification of $q\bar{q}$ -Meson Systems and Existence of New Scalar and Axial-Vector Mesons

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Abstract

The level classification of quark-antiquark systems is generally made by the LS -coupling scheme, resorting to the non-relativistic quark model. However, it has been well known that the π -meson with the $(L, S) = (0, 0)$ shows also the properties as a Nambu-Goldstone boson in the case of spontaneous breaking of a relativistic symmetry, the chiral symmetry. In this talk I present a covariant classification scheme for describing both the non-relativistic and the relativistic $q\bar{q}$ -mesons and point out possibility for existence of the new scalar meson nonet (to be assigned to the σ nonet) and also of the new axial-vector meson nonet, “chiralons,” which should be discriminated from the conventional 3P_0 and 3P_1 states, respectively.

[Introduction]

Recently strong evidences for existence of the light $\sigma(600)$ meson have been given by reanalyzing[1] the old $\pi\pi$ scattering phase shift data. The evidences[1] have been also given in the production processes. Reflecting these situations the σ has been revived in the lists of PDG'96 and '98 after missing over two decades. Furthermore, some evidences for existence of the $I = 1/2$ scalar meson $\kappa(900)$ have been also reported ¹ by reanalyzing[1] the $K\pi$ scattering phase shifts. These $\sigma(600)$ and $\kappa(900)$ mesons seem to be out of the conventional PDG-classification scheme based on the non-relativistic quark model(NRQM) and considered to be discriminated from the $q\bar{q}$ - 3P_0 states with the higher

¹ We have pointed out a possibility that these resonances together with the $a_0(980)$ and the $f_0(980)$ belong to the σ -nonet in the $SU(3)$ linear σ model. See, M. Y. Ishida in this workshop[2].

Table 1

Two contrasting viewpoints of $q\bar{q}$ mesons		
	Non-Relativistic	Relativistic
Model	Non R. quark Model	NJL Model
Approx. Symm.	LS coupling	Chiral symm.
quark mass	m_q^{const} large	m_q^{curr} small

masses. As is wellknown, there are the two contrasting viewpoints of $q\bar{q}$ -mesons(see Table I): the one is non-relativistic, based on the approximate symmetry of the LS -coupling in NRQM; while the other one is relativistic, based on the dynamically broken chiral symmetry in the NJL model. Now it is widely believed that the π meson (or π nonet) has a dual nature of a “non-relativistic” particle with the $(L, S) = (0, 0)$ and also of a relativistic particle as a Nambu-Goldstone boson with $J^P = 0^-$ in the case of spontaneous chiral symmetry-breaking. The purpose of this talk is to present a unification scheme of the above two viewpoints, giving covariant quark spin wave functions of the $q\bar{q}$ -mesons.

[Covariant extension of LS coupling scheme]

(*Boosted LS coupling scheme*) For many years we have developed the boosted LS coupling(bLS) scheme in the covariant oscillator quark model(COQM)[3] as a covariant extension of the LS coupling scheme in NRQM. The meson wave functions(WF) in COQM are tensors in $\tilde{U}(4) \times O(3, 1)$ space and reduce at the rest frame to those in the $SU(2)_{\text{spin}} \times O(3)_{\text{orbit}}$ -space in NRQM.

All $q\bar{q}$ -mesons are described unifiedly by the bilocal field $\Phi_A^B(x_1, x_2)$, where $x_1(x_2)$ is a space-time coordinate of a constituent quark (anti-quark), $A = (a, \alpha)$ ($B = (b, \beta)$) the flavor and Dirac spinor indices. The Φ fields are supposed to satisfy the bilocal Klein-Gordon equation of Yukawa with the squared-mass operator, and they are decomposed into the Fierz-components(, representing respective mesons,), that is, eigen states of \mathcal{M}^2 as

$$\Phi(X, x) = \sum_{P, n} (e^{iP_n \cdot X} \Psi_n(x; P_n) + e^{-iP_n \cdot X} \bar{\Psi}_n(x; P_n)) \quad (1)$$

$$\mathcal{M}^2(x_\mu, \frac{\partial}{\partial x_\mu}) \Psi_n(x; P_n) = M_n^2 \Psi_n(x; P_n) \quad (2)$$

The internal wave functions $\Psi(x; P_n)$ are factorized into the space-time portion f_n and the spinor one U_n ($\bar{U}_n \equiv -\gamma_4 U^\dagger \gamma_4$) as $\Psi_n = f_n(x, P_n) U(P)$, reflecting the general framework of bLS scheme. In COQM the f_n is expanded in terms of the covariant oscillator functions and the U is taken as a Bargmann-Wigner spinor.

The physical reason for validity of bLS scheme(in treating directly mass spectra instead of mass itself) comes from a phenomenological fact that the quark

binding potential is dominantly central: It is supported from the following successful results of applications of COQM: The prediction of the same form factor relations of weak semi-leptonic decays in the heavy-light quark system as obtained in the heavy-quark effective theory. The derivation of decay spectra of $B \rightarrow D/D^* l \nu$ in conformity with experiments. The well description of qualitative features of the radiative transitions of heavy quarkonium systems. (*Covariant spin WF of non-relativistic particle-BW spinor*) The spin WF, U , is assumed to satisfy the Bargmann-Wigner (BW) equation as

$$(iP \cdot \gamma^{(1)} + M)_\alpha{}^{\alpha'} U(P)_{\alpha'}{}^\beta = 0, \quad U(P)_\alpha{}^{\beta'} (-iP \cdot \gamma^{(2)} + M)_{\beta'}{}^\beta = 0, \quad (3)$$

where $P_\mu(M)$ is the four-momentum (mass) of the meson. The BW spinor is a covariant generalization of the Pauli spinor, and decomposed into the irreducible components as

$$U(P)_A{}^B = \frac{1}{2\sqrt{2}} [(-\gamma_5 P_s(P)_a{}^b + i\gamma_\mu V_\mu(P)_a{}^b)(1 + \frac{i\gamma_\mu P_\mu}{M})]_\alpha{}^\beta, \quad (4)$$

where $P_s(P)$ ($V_\mu(P)$) represents the pseudoscalar (vector) meson local field and $V_\mu(P)$ satisfies the Lorentz condition $P_\mu V_\mu = 0$. The mass of these (ground state) mesons is degenerate and taken as a simple sum of quark masses: $M \equiv m_1 + m_2$. Then Eq.(3) is easily seen to be equivalent to the following “free” constituent Dirac equations[3] (with a constraint on the momenta p_i)

$$(ip_1 \cdot \gamma^{(1)} + m_1)U(p_1, p_2) = 0, \quad U(p_1, p_2)(-ip_2 \cdot \gamma^{(2)} + m_2) = 0, \quad (5)$$

$$p_{1,2\mu} = \kappa_{1,2} P_\mu, \quad \kappa_1 + \kappa_2 = 1 \quad (\kappa_{1,2} = m_{1,2}/(m_1 + m_2)). \quad (6)$$

The constraint Eq.(6) implies that each constituent is in “parton-like motion”, and moving with the equal 3-dimensional velocity to that of total meson: That is, $\mathbf{v}_{1,2} = \frac{\mathbf{p}^{(1,2)}}{p_0^{(1,2)}} = \frac{\kappa_{1,2} \mathbf{P}_M}{\kappa_{1,2} P_{M,0}} = \frac{\mathbf{P}_M}{P_{M,0}} = \mathbf{v}_M$.

[Covariant spin WF of relativistic particle]

(*Covariant framework for description of $q\bar{q}$ meson system*) Our physical picture of the Yukawa bilocal function is given by the (B.S.) amplitude

$$\Phi_A{}^B(x_1, x_2) \approx \langle 0 | \psi_A(x_1) \bar{\psi}^B(x_2) | M \rangle. \quad (7)$$

Concerning space-time dependence the amplitude is Fourier-expanded as

$$\Phi_A{}^B(x_1, x_2) = \sum_{p_{1\mu}, p_{2\mu}} e^{ip_1 x_1} e^{ip_2 x_2} \Phi_A{}^B(p_1, p_2)$$

$$= \sum_{P_\mu = p_{1\mu} + p_{2\mu}} (e^{iPX} \Psi_A^B(x; P) + e^{-iPX} \bar{\Psi}_A^B(x; P)), \quad (8)$$

where in the last side is assumed that $p_{i,0} > 0$ and $P_0 = p_{1,0} + p_{2,0} > 0$. Concerning spin dependence the amplitude is² expanded by free bi-Dirac spinors $\overset{(i)}{W}(P)$ of constituent quarks and antiquarks as $(\langle A \rangle)$ denoting trace of A)

$$\Psi_\alpha^\beta(x, P) = \sum_i W_\alpha^{(i)\beta}(P) M^{(i)}(x, P), \quad M^{(i)}(x, P) \equiv \langle W^{(i)}(P) \Psi(x, P) \rangle, \quad (9)$$

and $\bar{\Psi}_\alpha^\beta$ is similarly represented by $\bar{W}_\alpha^{(i)\beta}$ and $M^{(i)\dagger} = \langle \bar{W}^{(i)} \bar{\Psi} \rangle$.
(*Covariant spin WF of relativistic particle*) First let's define the conventional positive and negative energy Dirac spinor by

$$\begin{aligned} \psi_\alpha(x) &= u_\alpha(p) e^{ipx}, \quad v_\alpha(p) e^{-ipx}, \quad p_\mu = (\mathbf{p}, iE), \quad E \equiv \sqrt{\mathbf{p}^2 + m^2} \\ (ip\gamma + m)u(p) &= 0, \quad (ip\gamma - m)v(p) = 0. \end{aligned} \quad (10)$$

As is easily seen from Eqs. (5) and (6), the bi-Dirac spinor for the non-relativistic particle is given by

$$W_\alpha^{(NR)\beta}(P) \equiv U_\alpha^\beta(P) = u_\alpha(p_1) \bar{v}^\beta(p_2)|_{p_i = \kappa_i P}. \quad (11)$$

On the othe hand, we choose as the bi-Dirac spinor for the relativistic particle

$$W_\alpha^{(R)\beta}(P) \equiv C_\alpha^\beta(P) = u_\alpha(p_1) \bar{u}^\beta(p_2)|_{p_i = \kappa_i P}, \quad (12)$$

which satisfies the “free” constituent Dirac equations

$$(ip_1 \cdot \gamma^{(1)} + m_1)C(p_1, p_2) = 0, \quad C(p_1, p_2)(ip_2 \cdot \gamma^{(2)} + m_2) = 0, \quad (13)$$

$$p_{1,2\mu} = \kappa_{1,2} P_\mu, \quad \kappa_1 + \kappa_2 = 1. \quad (14)$$

These equations (13) and (14) are equivalent to the new type of BW equation

$$(iP \cdot \gamma^{(1)} + M)C(P) = 0, \quad C(P)(iP \cdot \gamma^{(2)} + M) = 0. \quad (15)$$

The new BW spinor is decomposed into the irreducible components, the scalar S and axial-vector A_μ , as

² From here we shall consider only the ground states, or the “relativistic S -wave” states, neglecting the relative coordinates x_μ .

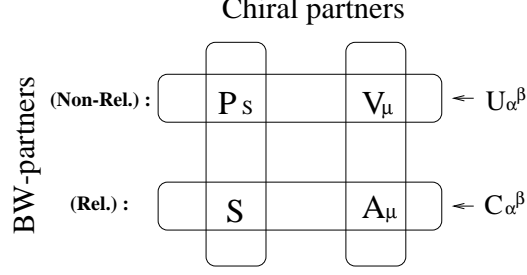


Fig. 1. Mutual relations among the composite ground state mesons

$$C(P)_A^B = \frac{1}{2\sqrt{2}} \left[\left(1 - \frac{i\gamma_\mu P_\mu}{M} \right) S(P)_a^b + \gamma_5 \gamma_\mu A_\mu(P)_a^b \left(1 - \frac{i\gamma_\mu P_\mu}{M} \right) \right]_\alpha^\beta, \quad (16)$$

where A_μ satisfies $P_\mu A_\mu(P) = 0$.

Here it is worthwhile to note that both the constituent quark and anti-quark, in this case also, are in “parton-like motion” and moving with the equal 3-dimensional velocity to that of total meson. Especially it is interesting that this situation on the anti-quark comes from (, noting $\bar{u}(p_2) \approx \bar{v}(-p_2)$,) its having anti-parallel motion to the meson and the negative energy, as

$$\mathbf{v}_2 = \frac{\mathbf{p}_2^{(2)}}{p_0^{(2)}} = \frac{-\kappa_2 \mathbf{P}_M}{-\kappa_2 P_{0,M}} = \frac{\mathbf{P}_M}{P_{0,M}} = \mathbf{v}_M.$$

[Chiral transformation of spin WF]

Since we know the chiral transformation property of the bi-Dirac spinors, $W_\alpha^\beta = U_\alpha^\beta$ or C_α^β , as $W \rightarrow W' = \exp(i\alpha\gamma_5)W\exp(i\alpha\gamma_5)$, we can directly deduce the transformation law

$$\begin{aligned} P'_s &= \cos 2\alpha P_s + \sin 2\alpha S; & V'_\mu &= V_\mu \\ S' &= -\sin 2\alpha P_s + \cos 2\alpha S; & A'_\mu &= A_\mu, \end{aligned} \quad (17)$$

for the composite mesons described by $P_s \propto \langle U i \gamma_5 \rangle$, $V_\mu \propto \langle U \gamma_\mu \rangle$, $S \propto \langle C 1 \rangle$, $A_\mu \propto \langle C i \gamma_5 \gamma_\mu \rangle$. Similarly we can derive the conventional transformation laws for mesons, appearing in any type of linear representations, for example, the $SU(2)(SU(3))$ linear σ model

$$\pi^{(i,r)} \propto \left\langle \frac{\tau^{(i)}}{2} \left(\frac{\lambda^{(r)}}{2} \right) i \gamma_5 U \right\rangle, \quad \sigma^{(0,r)} \propto \left\langle \frac{\tau^{(0)}}{2} \left(\frac{\lambda^{(r)}}{2} \right) C \right\rangle. \quad (18)$$

The mutual relations of these composite ground state mesons are schematically shown in Fig. 1.

[Experimental search for chiralons]

In the preceding sections we have shown the possibility of existence of a new scalar (possibly nonet) and a new axial-vector (possibly nonet), to be called

“chiralons,” as the “relativistic S -wave” states of composite $q\bar{q}$ meson systems. Since the former is considered already to be realized in nature as the σ -nonet[2], is naturally expected the existence of new axial-vector meson nonet, a_{1c} -nonet, which is to be the BW-partner of σ -nonet and also to be the chiral partner of ρ -nonet(see Fig. 1).

In the symmetric limit the mass of $I = 1$ member a_{1c} of the a_{1c} -nonet is equal to that of ρ . In the case of spontaneous breaking of the symmetry the famous relation had been predicted[4].

$$m_{a_{1c}} = \sqrt{2}m_{\rho} \approx 1.1 \text{ GeV}. \quad (19)$$

The experimental situation in search for the a_1 meson seems quite in confusion such that its mass and width look like to be variant, depending upon its production and decaying channels. We infer that this comes from the fact that there exists two axial-vector mesons with the same quantum numbers, a_1 and a_{1c} . [5]

It may be important to study experimentally on the existence of a_{1c} -nonet as well as of σ -nonet. We consider the experimental channels,

$$\begin{aligned} e^+ + e^- &\rightarrow \phi \text{ or } J/\psi \rightarrow \sigma \text{ or } a_{1c} + \cdots, \\ \gamma + \gamma &\rightarrow \sigma \text{ or } a_{1c} + \cdots, \end{aligned} \quad (20)$$

in VEPP are promising for this search.

References

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